**THEORY OF GROUP RINGS**

**AND ITS APLICATTIONS**

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**ABSTRACT**

Group Rings are developed from groups and rings, generating a new algebraic structure. In this paper connections between group rings and group and rings are made. After studying properties of group rings a very short study of correctors codes is made. Finally it is showed that there exists a relationship between cyclic codes and group algebras.

***Keywords:*** *algebra, groups, rings, codes.*

**BODY**

The paper is structured in six chapters. The first three have a similar structure, developing a theoretical description of each disciplines used for the application described in the fourth chapter.

In the first chapter the main concepts of algebra are developed, working harder in the main points to understand the theory of group rings.

In the second chapter group rings and group algebras are defined, showing that the second case is a especial one of the first case. Conditions for semisimplicity of a group rings are given in the Maschkes theorem.

The relationship between representations of a given group over a ring and modules over a group ring are given in the third chapter.

In the fourth chapter some algebraic elements are studied such as nilpotent elements, idempotent elements and torsion units.

In the fifth chapter a short introduction of units of group rings is given, showing that there are some recipes to build non trivial units.

Finally, in the sixth chapter it is presented a short introduction to code theory giving relevant importance to cyclic codes and showing that there exist a strong relationship between those kind of codes and group algebras.

**RESULTS**

Some highlights of this investigation are:

1. Let G be an abelian finite p-group, then G can be written as direct product of cyclic subgroups. This composition is unique.
2. RG is semisimply if and only if the following conditions are meet:

* R is a semisimple ring
* G is finite
* |G| is invertible in R.

1. There exists a bijection between representations of G over R and free RG-modules of finite rank.
2. If characteristic of the filed does not divide to the order of the group, then the study of cyclic codes is equivalent to the study of ideals in group algebras.

**CONCLUTIONS**

1. Para el estudio de los grupo-anillos es importante conocer la estructura de los grupos abelianos y hamiltonianos, así como la teoría de módulos y el teorema de Wedderburn-Artin.
2. Las condiciones necesarias y suficientes para que un grupo-anillo sea semisimple, vienen dadas por el teorema de Maschke.
3. Toda representación de un anillo conmutativo sobre un grupo dado,
4. corresponde a un módulo del grupo-anillo correspondiente.
5. En general no es fácil encontrar unidades no triviales en grupo-anillos, pero es posible construir algunas usando elementos idempotentes.
6. Las grupo-álgebras dan estructura matemática a los códigos correctores conocidos como c cíclicos.
7. The application of the mathematical formalism developed to thermodynamics and statistical mechanics on economics, through establish some basic similarities in the systems of these disciplines, can derive some interesting economic results.
8. Through this formalism was possible to introduce two economic variables usually not included in the classic study of this discipline: economic temperature and economic entropy as a measure of development and economic disorder respectively.
9. State function for the utility determined in the equation (1), was specified by the principal variables in the economic system from which we can derive the marginal utilities of goods and money and their relationship with the utility, was determined individually with the development and disorder in the economy.
10. Maxwell relations in the economy, determined in equation (2), can provide a mechanism to measure variations in temperature and entropy in economic, function of other variables whose measurement were determined feasible.
11. The Gibb -Duhem relation in the economy determined in the equation (3), allows us to relate changes in the intensive variables in the economic system. This relationship is evident as the decline in prices of goods and money making the economy temperature increase resulting in greater economic development .
12. Finally, an expression that shows the distribution of individuals in the economic system with reference to the amount of wealth of these, shown in the equation (4). In this expression a pyramidal shape for this distribution is established , as shown by most economies .

**RECOMMENDATIONS**

1. Considering flexibility in some assumptions made in this study, relationships found can be verified through some kind of econometric study.
2. Is possible consider other similarities between other branches of physics and economics in order to compare the results between these applications.
3. Results obtained in this work are only part of which can be obtained based on the proposed analogies, that development can be made ​​widespread. For example, is possible to study the case where the economy is treated as an open system, which is a more realistic situation, and might be more appropriate to continue working with the statistical mechanics, specifically with other ensembles untreated.

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To God and my family.

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